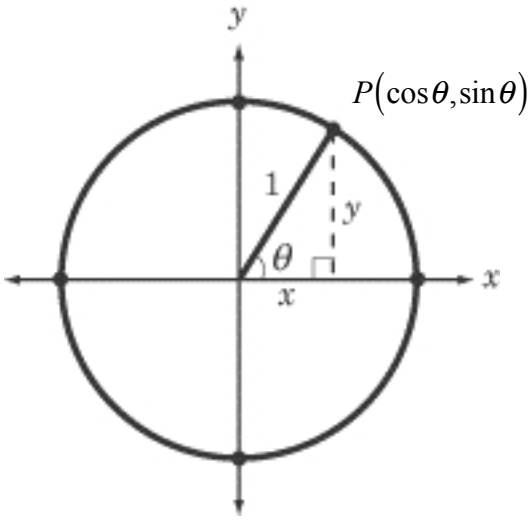
Pythagorean Identities

These notes are intended as a companion to section 7.4 (p. 621 – 625) in your workbook. You should also read the section for more complete explanations and additional examples.

Pythagorean Identities

Recall that the coordinates of a point *P* on the unit circle can be written as:



If we apply the Pythagorean theorem to the triangle in the diagram, we can observe the following relationship:

$$x^2 + y^2 = 1$$
$$\left(\cos\theta\right)^2 + \left(\sin\theta\right)^2 = 1$$

$$\cos^2\theta + \sin^2\theta = 1$$

If we divide every term in this identity by $\cos^2\theta$ we get the following result:

$$\cos^2\theta + \sin^2\theta = 1$$
$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

Similarly, if we divide by $\sin^2\theta$ we get the following result:

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\cot^2\theta + 1 = \csc^2\theta$$

These three identities are known as the **Pythagorean identities**.

$$\cos^2\theta + \sin^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

These can be used to prove other identities or to simplify an equation before solving it.

Strategies for Proving Identities

1. Start with the more complicated side. Try to reduce it to the simpler side.
2. If the first strategy doesn't work, try simplifying each side separately to "meet in the middle."
3. Combine terms where possible.
4. Combine fractions to get a common denominator.
5. Multiply and divide where possible. (e.g. FOIL)
6. Cancel common factors.
7. Factor expressions if possible. (e.g. difference of squares)
8. Rewrite everything in terms of sine and cosine.
9. Try multiplying by 1. e.g. $\frac{\sin x}{\sin x}$

Example 1 (sidebar p. 623)
Prove each identity.

a) $\cot\theta + \tan\theta = \csc\theta\sec\theta$

b) $\cot^3\theta = \cot\theta\csc^2\theta - \cot\theta$

Example 2 (sidebar p. 624)
Prove each identity.

a) $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

b) $\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}=2\csc^2\theta$

Example 3 (sidebar p. 625)

Use algebra to solve the equation $3-3\cos x-2\sin^2 x=0$ over the domain $0\le x\le\frac{3\pi}{2}$.

Example 4 (not in workbook)
Prove the identity:

$$\tan^4\theta - \sec^4\theta = -\tan^2\theta - \sec^2\theta$$

Homework: #3, 5, 6ii, 7ii, 9ii, 10 in the exercises (p. 626 – 632). Answers on p. 633.